XLVI. *Problems by* Edward Waring, *M A.* *and* Lucaſian *Profeſſor of Mathematics in the Univerſity of* Cambridge, *F. R. S.*

P R O.

Read April 21, 1763.

1. Invenire, quot radices impoſſibiles habet data biquadratica æquatio $x^4 + qx^2 - rx + s = 0$.

1^mo^ Sit $256\,s^3 - 128\,q^2 s^2 + \overline{144\,r^2 q + 16\,q^4} \times s - 27\,r^4 - 4\,r^2 q^3$ negativa quantitas, & duas & non plures impoſſibiles radices habet data æquatio.

2^do^ Sit affirmativa quantitas, & vel $-q$ vel $q^2 - 4\,s$ negativa quantitas, & datæ æquationis quatuor radices erunt impoſſibiles.

3^tio^. Sit nihilo æqualis, & vel $-q$ vel $q^2 - 4\,s$ negativa quantitas, & datæ æquationis duæ inæquales radicis erunt impoſſibiles.

2. Invenire, quot radices impoſſibiles habet data æquatio $x^5 + q\,x^3 - r\,x^2 + s\,x - t = 0$.

1^mo^ Si ſigna terminorum æquationis $w^{10} + 10\,q\,w^9 + \overline{39\,q^2 + 10\,s} \times w^8 + \overline{80\,q^3 + 50\,q\,s + 25\,r^2} \times w^7 + \overline{95\,q^4 + 124\,q^2 s - 95\,s^2 + 92\,q\,r^2 + 200\,r\,t} \times w^6 + \overline{66\,q^5 - 360\,q\,s^2 + 196\,q^3 s + 118\,q^2 r^2 + 260\,r^2 s + 625\,t^2 + 400\,q\,r\,t} \times w^5 + \overline{25\,q^6 + 40\,s^3 - 53\,r^4 + 52\,q^3 r^2 - 522\,q^2 s^2 + 194\,q^4 s + 708\,q\,r^2 s + 240\,q^2 r\,t + 1750\,q\,t^2 - 950\,s\,r\,t} \times w^4 + \overline{4\,q^7 + 106\,q^5 s - 80\,q\,s^3 - 308\,q^3 s^2 - 102\,q\,r^4 - 7\,q^4 r^2 + 570\,r^2 s^2 + 612\,q^2 r^2 s + 700\,r^3 t - 3750\,t^2 s + 2500\,t^2 q^2 + 80\,r\,t\,q^3 - 2150\,q\,r\,s\,t} \times w^3 + 400\,s^4 - 360\,q^2 s^3 - 15\,q^4 s^2 + 24\,q^6 s - 8\,q^5 r^2$

— 45

$-45 q^2 r^4 - 270 r^4 s + 140 r^2 s q^3 + 960 r^2 s^2 q + 1875 t^2 r^2 + 1000 t r s^2 - 5000 t^2 q s + 1750 t^2 q^3 + 40 t r q^4 + 600 t r^3 q - 1650 t r s q^2 \times w^2 + 36 q^5 s^2 - 224 q^3 s^3 + 320 q s^4 + 4 q^3 r^4 + 27 r^6 - 40 r^2 s^2 + 434 r^2 q^2 s^2 - 24 r^2 s q^4 - 198 r^4 q s + 5000 t^2 s^2 - 450 t r^3 s - 6250 t^3 r + 675 t^2 q^4 - 3750 t^2 q^2 s + 3000 t^2 r^2 q + 60 t r^3 q^2 + 200 t r s^2 q - 330 t r q^3 s \times w + 3125 t^4 - 3750 q r t^3 + 2000 s^2 q + 2250 r^2 s - 900 s q^3 + 825 r^2 q^2 + 108 q^5 \times t^2 - 1600 s^3 r - 560 r q^2 s^2 - 16 r^3 q^3 + 630 r^3 q s + 72 r s q^4 - 108 r^5 \times t + 256 s^5 - 128 q^2 s^4 + 144 r^2 q s^3 + 16 q^4 s^3 - 27 r^4 s^2 - 4 r^2 q^3 s^2 = 0$. continuo mutentur de $+$ in $-$; & $-$ in $+$; nullas impoſſibiles radices habet data æquatio.

2do. Si ſigna terminorum æquationis haud continuo mutentur de $+$ in $-$ & $-$ in $+$; duæ vel quatuor datæ æquationis radices erunt impoſſibiles, prout ultimus ejus terminus ſit negativa vel affirmativa quantitas.

3tio. Si ultimus ejus terminus nihilo ſit æqualis, & ſigna terminorum æquationis haud continuo mutentur de $+$ in $-$ & $-$ in $+$; tum vel quatuor vel duæ radices datæ æquationis erunt impoſſibiles, prout duo & non plures ultimi datæ æquationis termini nihilo ſint æquales, necne.

P R O.

Sint x, y, v, abſciſſa, ordinata & area datæ curvæ, & ſit $y^n + \overline{a+bx} \times y^{n-1} + \overline{c+dx+ex^2} \times y^{n-2} + \overline{f+gx+hx^2+kx^3} \times y^{n-3} +$ &c. $= 0$: invenire, utrum area (v) quadrari poteſt, necne.

Supponamus æquationem ad aream eſſe $v^n + \overline{A+Bx+Cx^2}\, v^{n-1} + \overline{D+Ex+Fx^2+Gx^3+Hx^4} \times$

$v^{n-2} + \overline{I + Kx + Lx^2 + Mx^3 + Nx^4 + Ox^5 + Px^6}$
$\times v^{n-3} +$ &c. $= 0$. & consequenter erit $nyv^{n-1} + \overline{n-1}$
$\overline{A + Bx + Cx^2}yv^{n-2} + \overline{n-2} \times \overline{D \times Ex + Fx^2 + Gx^3 + Hx^4}$
$\overline{B + 2Cx}\; v^{n-1} + \quad \overline{E + 2Fx + 3Gx^2 + 4Hx^3}$

$$\left.\begin{array}{l} \times\, yv^{n-3} + \text{\&c.} \\ \times\, v^{n-2} + \text{\&c.} \end{array}\right\} = 0.$$

Ex quibus æquationibus, si methodis notis extermi-netur (v), habebimus æquationem, quæ exprimit relationem inter (x) & (y). Hujus autem æquationis coefficientes æquari debent coefficientibus datæ æquationis $y^n + \overline{a + bx}\, y^{n-1} + \overline{c + dx + ex^2} + y^{n-2} +$ &c. $= 0$; & si quantitates A, B, C, &c. exinde determinari possunt, curva quadratur, est enim $v^n + \overline{A + Bx + Cx^2} \times v^{n-1} + \overline{D + Ex + Fx^2 + Gx^3 + Hx^4} \times v^{n-2} +$ &c. $= 0$; aliter autem quadrari non potest.

Ex. Sit data æquatio $y^2 + x^2 - 1 = 0$, & supponamus æquationem ad aream $v^2 + D + Ex + Fx^2 + Gx^3 + Hx^4 = 0$; & erit $2vy + E + 2Fx + 3Gx^2 + 4Hx^3 = 0$, ita reducantur hæ duæ æquationes in unam, ut exterminatur (v), & resultat æquatio $y^2 + \dfrac{16H^2x^6 + 24HGx^5 + \overline{16HF + 9G^2}x^4 + \overline{8EH + 12FG}x^3 + \overline{6GE + 4F^2}x^2 + 4FEx + E^2}{4 \times \overline{Hx^4 + Gx^3 + Fx^2 + Ex + D}} = 0$; debet autem fractio $\dfrac{16H^2x^6 + 24HGx^5 + \overline{16HF + 9G^2}x^4 + \overline{8EH + 12FG}x^3 + \overline{6GE + 4F^2}x^2 + 4FEx + E^2}{4 \times \overline{Hx^4 + Gx^3 + Fx^2 + Ex + D}}$ esse $x^2 - 1$; & consequenter

$$\begin{aligned} 4H &= 16H^2 \\ 4G &= 24HG \\ 4F - 4H &= 16HF + 9G^2 \\ 4E - 4G &= 8HE + 12FG \\ 4D - 4F &= 6GE + 4F^2 \\ -4E &= 4FE \\ -4D &= E^2 \end{aligned}$$

ſed e methodo communes diviſores inveniendi conſtat has æquationes inter ſe contradictorias eſſe, & conſequenter curvam haud generaliter eſſe quadrabilem.

THEO.

Sint x, y, v, abſciſſa & ordinatæ curvarum ABCD EFGHI &c. & $A\beta\gamma\delta\varepsilon$ &c. & ſit $y = px^n$, & $v =$

$$\frac{n}{2.3} pa^{n-1}x - \frac{n \times \overline{n-1} \times \overline{n-2}}{30 \times 2 \times 3} pa^{n-3}x^3 + \frac{n \times \overline{n-1} \times \overline{n-2} \times \overline{n-3} \times \overline{n-4}}{42 \times 2 \times 3 \times 4 \times 5} pa^{n-5}x^5 - \frac{n \times \overline{n-1} \times \overline{n-2} \times \overline{n-3} \times \overline{n-4} \times \overline{n-5} \times \overline{n-6}}{30 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7} pa^{n-7}x^7 + \frac{5n \times \overline{n-1} \times \overline{n-2} \times \overline{n-3} \times \overline{n-4} \times \overline{n-5} \times \overline{n-6} \times \overline{n-7} \times \overline{n-8}}{66 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9} pa^{n-9}x^9 - \frac{691 \times n \times \overline{n-1} \times \overline{n-2} \times \overline{n-3} \times \overline{n-4} \times \overline{n-5} \times \overline{n-6} \times \overline{n-7} \times \overline{n-8} \times \overline{n-9} \times \overline{n-10}}{2730 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11} pa^{n-11}$$

$x^{11} +$ &c. cujus ultimus terminus debet eſſe x^{n-1} vel x^{n-2}, prout (n) eſt par vel impar numerus.

Sit $x = AP = a$, biſecetur AP in T in duas æquales partes, & ducatur linea $ET\delta$, & ſi AE, EM, AM, jungantur; erit triangulum AEM $=$ TP $\varepsilon\delta$ T areæ.

Deinde,

Deinde, bisecentur T P, A T in R and V, & ducantur R G, C V γ; & jungantur A C, C E, E G, G M; & erunt duo triangula A C E + E G M = V T $\delta \gamma$ V areæ.

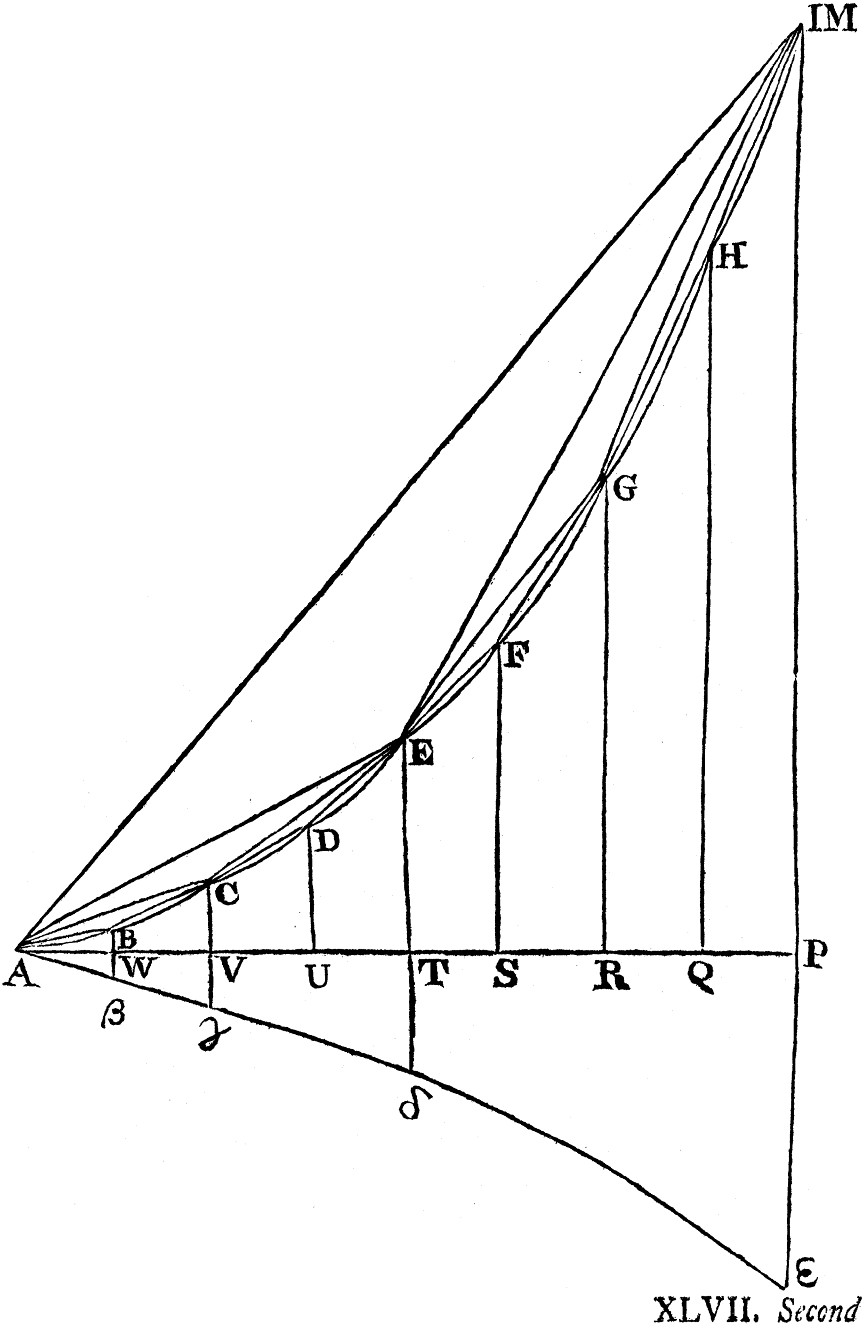
Eodem modo, si partes A V, V T, T R, R P iterum bisecentur in W, U, S, Q, & ducantur lineæ B W β, U D, S F, Q H; & jungantur A B, B C, C D, D E, E F, F G, G H, H M; erunt quatuor triangula A B C + C D E + E F G + G H M = W V $\gamma \beta$ W areæ; & sic deinceps.

Cor. 1. Si curva A B C & M sit conica parabola, $(c, e)\, y = p i x^2$, erit $v = \frac{1}{3} p a x$; & A $\beta \gamma \delta$ &c. erit recta linea; & propositio eadem est cum notissimâ propositione Archimedis de quadraturâ parabolæ.

Cor. 2. Si $y = p x^3$, erit $v = \frac{1}{2} p a^2 x$, & A $\beta \gamma \delta$ &c. iterum recta linea.

Cor. 3. Datâ curvâ, cujus æquatio est $y = p x^{2n}$, inveniri potest altera curva, cujus dimensiones sunt $(2n - 1)$, in quâ summæ triangulorum ad singulas bisectiones erunt respectivè æquales summis triangulorum datæ curvæ.

His adjici potest, quod si loco bisectionis abscissa A P aliâ quâvis ratione in æquales partes dividatur, summæ triangulorum curvæ A B C D &c. ad singulas divisiones æquales erunt segmentis curvæ A $\beta \gamma \delta$ &c.



XLVII. *Second*